Sprawdzian na półmetku – rozwiązania

A

zad.1 (3p)

$9^{-\frac{1}{4}} \cdot \left(\frac{1}{3}\right)^{\frac{-3}{4}} : \sqrt[6]{27} = 3^{-\frac{1}{2}} \cdot 3^{\frac{3}{4}} : 3^{\frac{1}{2}} = 3^{-\frac{1}{4}}$

Zad.2 (3p)

n = 2k +1 lub n= 2k – 1

$(2k + 1)^2 – 1 = 4k^2 + 4k = 4k(k+1)$ lub $(2k - 1)^2 – 1 = 4k^2 - 4k = 4k(k - 1)$

i uzasadnienie

Zad.3 (4p)

$-3(x – 2) - 5x < -10$

$-3x + 6 - 5x < -10$

$-8x<-16$

$x>2$

$x \in (2, +\infty)$

Zad.4 (3p)

0= -a + b i -20 = 9a + b

a= -2 i b=-2

y = -2x -2

zad.5 (6p)

$7^2 + (c-1)^2 = c^2$

$49 + c^2 -2c + 1 = c^2$

-2c = - 50

c =25 d=0,5c=12,5

P = 0,5·7·24=84 84 = 0,5·25h $h = 6\frac{18}{25}$

zad.6 (3p)

$|x + 5| \leq 4|x + 5| - 6$

$-3|x + 5| \leq -6$

$|x + 5| \geq 2$

$x \in (-\infty, - 7] \cup [-3,+\infty)$

zad.7 (6p)

$y = a(x+8)(x-2)$ $\quad y = -\frac{1}{4}(x+8)(x-2)$

$6 = a \cdot 4 \cdot (-6)$ $\quad y = -\frac{1}{4}(x^2-2x+8x-16) = -\frac{1}{4}x^2 - \frac{3}{2}x + 4$

$a = -\frac{1}{4}$

$p = -3$ $\quad q = \frac{25}{4}$ $\quad y = -\frac{1}{4}(x+3)^2 + \frac{25}{4}$

$x \in [-8, 2]$

zad.8 (6p)

$x^2 = 15^2 + 12^2 - 2 \cdot 15 \cdot 12 \cdot \cos 60^o$

$x^2 = 189$ i $x > 0$

$x = 3\sqrt{21}$

$12 < 3\sqrt{21} < 15$ $\quad \frac{12}{sin\alpha} = \frac{3\sqrt{21}}{sin60^0}$ $\quad 12\frac{\sqrt{3}}{2} = 3\sqrt{21}\sin\alpha$ $\quad \sin\alpha = \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$

zad.9 (6p)

$p = (13+20+21):2 = 27$

$P = \sqrt{27 \cdot 14 \cdot 7 \cdot 6} = 3 \cdot 3 \cdot 2 \cdot 7 = 126$

$126 = 27 \cdot r$ $\quad r = \frac{14}{3} = 4\frac{2}{3}$

$126 = \frac{13 \cdot 20 \cdot 21}{4R}$ $\quad R = \frac{65}{6} = 10\frac{5}{6}$

$126 = 0{,}5 \cdot 13 \cdot 20 \cdot \sin\alpha$ $\quad \sin\alpha = \frac{126}{130} = \frac{63}{65}$

zad.10 (4p)

$9x^3 - 18\sqrt{2}x^2 - 4x + 8\sqrt{2} = 0$ $\quad 9x^2(x - 2\sqrt{2}) - 4(x - 2\sqrt{2}) = 0$

$(x - 2\sqrt{2})(9x^2 - 4) = 0$ $\quad (x - 2\sqrt{2})(3x - 2)(3x + 2) = 0$

$x = 2\sqrt{2}$, $x = \frac{2}{3}$, $x = -\frac{2}{3}$

B

zad.1 (3p)

$16^{-\frac{1}{4}} \cdot \left(\frac{1}{4}\right)^{\frac{-3}{4}} : \sqrt[6]{64} = 4^{-\frac{1}{2}} \cdot 4^{\frac{3}{4}} : 4^{\frac{1}{2}} = 4^{-\frac{1}{4}} = 2^{-\frac{1}{2}}$

Zad.2 (3p)

n = 2k +1 lub n= 2k – 1

$(2k + 1)^2 + 1 = 4k^2 + 4k+2 = 2(2k^2+2k+1)$ lub $(2k - 1)^2 + 1 = 4k^2 - 4k+2 = 2(2k^2-2k + 1)$ i uzasadnienie

Zad.3 (4p)

$5(-x – 2) - 3x < 6$

$-5x - 10 - 3x < 6$

$- 8x < 16$

$x > -2$

$x \in (-2, +\infty)$

Zad.4 (3p)

0= -2a + b i -10 = 3a + b

a= -2 i b=-4

y = -2x -4

zad.5 (6p)

$5^2 + (c-1)^2 = c^2$

$25 + c^2 -2c + 1 = c^2$

$-2c = - 26$

c =13 d=0,5c=6,5

P = 0,5·5·12=30 30 = 0,5·13h $h = 4\frac{8}{13}$

zad.6 (3p)

$4|x+7| \leq |7+x| - 3$

$3|x + 7| \leq -3$

$|x + 5| \leq -1$

Nierówność sprzeczna

zad.7 (6p)

$y = a(x+6)(x-4)$ $\quad y = -\frac{1}{3}(x+6)(x-4)$

$8 = a\cdot 4\cdot(-6)$ $\quad y = -\frac{1}{3}(x^2-4x+6x-24) = -\frac{1}{3}x^2 - \frac{2}{3}x + 8$

$a = -\frac{1}{3}$

$p = -1$ $\quad q = 8\frac{1}{3}$ $\quad y = -\frac{1}{4}(x+1)^2 + 8\frac{1}{3}$

$x \in (-\infty, -6] \cup [4, +\infty)$

zad.8 (6p)

$x^2 = 16^2 + 20^2 - 2\cdot 16\cdot 20\cdot \cos 60^o$

$x^2 = 336$ i $x>0$

$x = 4\sqrt{21}$

$16 < 4\sqrt{21} < 20$ $\quad \frac{16}{sin\alpha} = \frac{4\sqrt{21}}{sin60^0}$ $\quad 16\frac{\sqrt{3}}{2} = 4\sqrt{21}\sin\alpha$ $\quad \sin\alpha = \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$

zad.9 (6p)

$p = (10+17+21):2 = 24$

$P = \sqrt{24\cdot 14\cdot 7\cdot 3} = 4\cdot 3\cdot 7 = 84$

$84 = 24\cdot r$ $\quad r = \frac{7}{2} = 3\frac{1}{2}$

$84 = \frac{10\cdot 17\cdot 21}{4R}$ $\quad R = \frac{85}{8} = 10\frac{5}{8}$

$84 = 0{,}5\cdot 17\cdot 10\cdot \sin\alpha$ $\quad \sin\alpha = \frac{84}{85}$

zad.10 (4p)

$9x^3 - 27\sqrt{3}x^2 - 4x + 12\sqrt{3} = 0$ $\quad 9x^2(x - 3\sqrt{3}) - 4(x - 3\sqrt{3}) = 0$

$(x - 3\sqrt{3})(9x^2 - 4) = 0$ $\quad (x - 3\sqrt{3})(3x-2)(3x+2) = 0$

$x = 3\sqrt{3}$, $x = \frac{2}{3}$, $x = -\frac{2}{3}$